$F(S)$

$$\frac{1}{S-1} = AS^3 + BS^2 + CS + D + \frac{ES^4}{(S-1)}$$

$$(S-1)^{-1} = AS^3 + BS^2 + CS + \cancel{D}$$

$$\boxed{-(S-1)^{-2} = 3AS^2 + 2BS} + C$$

$S=0$

$$\boxed{-1 = C}$$

$$\boxed{2(S-1)^{-3} = 6AS} + 2B$$

$S=0$

$$-2 = 2B$$

$$\boxed{B = -1}$$

$$-6(S-1)^{-4} = 6A$$

$$-6 = 6A$$

$$\boxed{A = -1}$$

$$-\frac{1}{s}-\frac{1}{s^2}-\frac{1}{s^3}-\frac{1}{s^4}+\frac{1}{(s-1)}$$

$$\cancel{-1-t-t^2-t^3+}$$

$$\frac{2!}{s^3}\cdot\frac{1}{2}$$

$$\frac{3!}{s^3}\cdot\frac{1}{6}$$

$$\cancel{-t-t^2-\frac{1}{2}t^3-\frac{1}{6}t^4+e^t}$$

$$\boxed{-1-t-\frac{1}{2}t^2-\frac{1}{6}t^3+e^t}$$

$$\mathcal{L}^{-1}\left\{\frac{s^3-s^2-2s-1}{s^4-s^2-2}\right\}$$

$$\frac{s^3-s^2-2s-1}{(s^2-2)(s^2+1)} = \frac{As+B}{(s^2-2)} + \frac{Ds+E}{(s^2+1)}$$

$$s^3-s^2-2s-1 = (As+B)(s^2+1) + Ds+E(s^2-2)$$

$$s=\sqrt{2}$$

$$\cancel{2}^{3/2}-2-2\sqrt{2}-1 = (A\sqrt{2}+B)(3)$$

$$\cancel{(-2)^{3/2}}-2+2\sqrt{2}-1 = (-A\sqrt{2}+B)(3)$$

$$-4-2=6B$$

$$\boxed{B=-1} \qquad \boxed{A=0}$$

$-i + 1 - 2i - 1 = -3(Di + E)$

$-3i = \; = -3Di \oplus -3E$

$-3 = -3D$

$\boxed{E = 0}$

$\boxed{D = 1}$

$$\frac{-1}{s^2 - 2} + \frac{s}{(s^2+1)}$$

$\cos(t)$

$$\frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{s^2 - 2} + \frac{s}{s^2+1}$$

$$\boxed{\frac{-1}{\sqrt{2}} \operatorname{senh}(\sqrt{2}t) + \cos(t)}$$

$\mathcal{L}\{t^2 e^{4t}\}$

$= \mathcal{L}\{t^2\}\Big|_{s \to s-4}$

$= \dfrac{2}{s^3} = \boxed{\dfrac{2}{(s-4)^3}}$

$\mathcal{L}\{5t^6 e^{-2t}\}$

$\mathcal{L}\{5t^6\}\Big|_{s \to s+2}$

$\dfrac{3600}{s^7}\Big|_{s+2}$

$\boxed{\dfrac{3600}{(s+2)^7}}$

$\mathcal{L}\left[(t+e^{-t})^2\right]$

$t^2 + t2e^{-t} + e^{-2t}$

$$\frac{2}{s^3} + \frac{2}{s^2}\Big|_{s+1} + \frac{1}{s+2}$$

$$\boxed{\frac{2}{s^3} + \frac{2}{(s+1)^2} + \frac{1}{s+2}}$$

$e^{3t} \quad \cosh(2t)$

$$\mathcal{L}\cosh\Big|_{s-3} = \frac{s}{s^2-4}$$

$$\boxed{\frac{s-3}{(s-3)^2-4}}$$

$$\mathcal{L}\{e^{-2t}\,\text{sen}\,5t\}$$

$$\mathcal{L}\{\text{sen}\,5t\}\Big|_{s\to s+2}$$

$$\frac{5}{s^2+25} = \boxed{\frac{5}{(s+2)^2+25}}$$

$$\mathcal{L}\{t\,\cosh(3t)\}$$

$$\frac{1}{2}\,\mathcal{L}\left(t e^{3t} + t e^{-3t}\right)$$

$$\frac{1}{2}\left(\frac{1}{s^2}\Big|_{s\to s-3} + \frac{1}{s^2}\Big|_{s\to s+3}\right)$$

$$\frac{1}{2}\left[\frac{s}{(s-3)^2} + \frac{1}{(s+3)^2}\right]$$

TALLER 10

①

a) $f(t) = \begin{cases} -2 & \text{si } 0 \leq t < 2 \\ 2 & \text{si } 2 \leq t < 3 \\ -4 & 3 \leq t \end{cases}$

$$F(s) = -2\,(\cancel{U_0(t)} - U_2(t)) + 2\,(U_2(t) - U_3(t)) - 4\,(U_3(t))$$

$$-2 + 2U_2(t) + 2U_2(t) - 2U_3(t) - 4U_3(t)$$

$$\mathcal{L}\{4U_2(t) - 6U_3(t) - 2\}$$

$$\boxed{F(S) = \frac{4e^{-2S}}{S} - \frac{6e^{-3S}}{S} - \frac{2}{S}}$$

②

b) $f(t) = \begin{cases} 0 & 0 \leq t \leq \pi \\ t - \pi & \pi \leq t < 2\pi \\ 0 & 2\pi \leq t \end{cases}$

$$(t-\pi)\,(U_\pi - U_{2\pi})$$

$$(t-\pi)\,U_\pi - t - \pi\; U_{2\pi}$$

$$e^{-\pi s}\,\mathcal{L}\{t+\pi-\pi\} - e^{-2\pi}\,\mathcal{L}\{t+2\pi-\pi\}$$

$$\boxed{\frac{e^{-\pi s}}{s^2} - e^{-2\pi}\left(\frac{1}{s^2} + \frac{\pi}{s}\right)}$$

c) $f(t) = \begin{cases} 0 & 0 \le t < 1 \\ t^2 - 2t + 2 & 1 \le t \end{cases}$

$(t^2 - 2t + 2)(U_1(t))$

$e^{-s} \mathcal{L}\{(t+1)^2 - 2(t+1) + 2\}$

$e^{-s} \mathcal{L}\{t^2 + \cancel{2t} + 1 - \cancel{2t} \cancel{-2} \cancel{+2}\}$

$e^{-s} \mathcal{L}\{t^2 + 1\}$

$\boxed{e^{-s}\left(\frac{2}{s^3} + \frac{1}{s}\right)}$

d) $f(t) = e^t \cos(t) + (1 + e^{2t})^2 - 5e^{-t} \cosh(t)$

$\mathcal{L}\{f(t)\} = F(s)$

$F(s) = \frac{s}{s^2+1}\Big|_{s=s-a} + \mathcal{L}\{1 + 2e^{2t} + e^{4t}\} + \frac{5s}{s^2-1}\Big|_{s=s-a}$

$F(s) = \frac{s-1}{(s-1)^2+1} + \frac{1}{s} + \frac{2}{s-2} + \frac{1}{s-4} + \frac{5(s+1)}{(s+1)^2-1}$

e) $f(t) = \operatorname{sen}(3t) \cos(3t)$

$\operatorname{sen} 2\theta = 2 \operatorname{sen}\theta \cdot \cos\theta$

$\frac{\operatorname{sen}(2(3t))}{2}$ $\frac{\operatorname{sen} 6t}{2}$

$\frac{1}{2} \mathcal{L}\{\text{sen}(6t)\}$

$\frac{1}{2} \quad \frac{6}{s^2+36}$

$\boxed{\frac{3}{s^2+36}}$

(f) $\cos^2 t + 7\ \text{sen}(3t - \pi/3)$

$\frac{1+\cos(2t)}{2} + 7\left[\text{sen}(3t)\cos(\pi/3) - \text{sen}\,\pi/3 \cdot \cos(3t)\right]$

$\mathcal{L}\{f(t)\} = F(s)$

$F(s) = \frac{1}{2s} + \frac{1}{2}\frac{s}{s^2+4} + \frac{1}{2}\frac{3}{s^2+9} - \frac{\sqrt{3}}{2}\frac{s}{s^2+9}$

(g) $f(t) = e^{-2t}\cos(4t)$

$\mathcal{L}\{f(t)\} = F(s)$

$F(s) = \frac{s}{s^2+16}\Big|_{s=s+2}$

$F(s) = \frac{s+2}{(s+2)^2+16}$

(h) $f(t) = e^{7t} \cdot 4t$

$\mathcal{L}\{f(t)\} = F(s)$

$F(s) = \frac{4}{s^2}\Big|_{s=s-9}$

$F(s) = \frac{4}{(s-7)^2}$

(I) $f(t) = e^{2t}(t^2+2)^2$

$\mathcal{L}\{f(t)\} = F(S)$

$F(S) = e^{2t}(t^4 + 4t^2 + 4)$

$$\frac{24}{S^5}\Big|_{S-2} + 4\,\frac{2}{S^3}\Big|_{S-2} + \frac{4}{S}\Big|_{S-2}$$

$$F(S) = \frac{24}{(S-2)^2} + \frac{4\cdot 2}{(S-2)^3} + \frac{S}{(S-2)}$$

(J) $f(t) = (t-3)\,U_2(t) - (t-2)\,U_3(t)$

$\mathcal{L}\{f(t)\} = F(S)$

$e^{-2S}(t-3) - e^{-3S}(t-2)$

$F(S) = e^{-2S}\,\mathcal{L}\{t+2-3\} - e^{-3S}(t+3-2)$

$$F(S) = e^{-2S}\left(\frac{1}{S^2} - \frac{1}{S}\right) - e^{-3S}\left(\frac{1}{S^2} + \frac{1}{S}\right)$$

(K) $f(t) = \cos(2t)\,U_\pi(t)$

$\mathcal{L}\{f(t)\} = F(S)$

$F(S) = e^{-\pi S}\cos(2(t+\pi))$

$F(S) = e^{-\pi S}\cos(2t + 2\pi)$

$= e^{-\pi S}\,\mathcal{L}\{\cos(2t)\}$

$$= e^{-\pi S}\,\frac{S}{S^2+4}$$

~~$\cos 2t \cdot \text{sen}\, 2\pi - \cos(2\pi)$~~

$\cos(2t)\cdot\cos(2\pi)$ 1

$- \text{sen}(2t)\cdot\text{sen}\,2\pi$ 0

(L) ~~f(t)=cos t~~ $f(t) = (2t+1)\, U_1(t)$

$\mathcal{L}\{f(t)\} = F(s)$

$F(s) = e^{-s}\ (2(t+1)+1)$

$F(s) = e^{-s}\ \mathcal{L}\{2t+2+1\}$

$F(s) = e^{-s}\ \mathcal{L}\{2t+3\}$

$F(s) = e^{-s}\left(\frac{2}{s^2} + \frac{3}{s}\right)$

(M) $f(t) = \cosh(t) \cdot U_\pi(t)$

$\mathcal{L}\{f(t)\} = F(s)$

$F(s) = e^{-\pi s}\ \mathcal{L}\{\cosh(t+\pi)\}$

$F(s) = e^{-\pi s}\ \mathcal{L}\left\{\frac{e^t \cdot e^\pi + e^{-t} \cdot e^{-\pi}}{2}\right\}$

$F(s) = \frac{e^{-\pi s}}{2}\left(\frac{e^\pi}{s-1} + \frac{e^{-\pi}}{s+1}\right)$

(2)

(a) $f(t) = \begin{cases} 0 & 0 \le t < 3\pi/2 \\ \operatorname{sen}(t) & 3\pi/2 \le t \end{cases}$

$f(t) = \operatorname{sen}(t)\ U_{\frac{3\pi}{2}}$

$\mathcal{L}\{f(t)\} = F(s)$

$$F(S) = e^{-\frac{3\pi s}{2}} \mathcal{L}\left\{ \operatorname{sen}\left(t + \frac{3\pi}{2}\right) \right\}$$

$$F(S) = e^{-\frac{3\pi s}{2}} \mathcal{L}\left\{ \operatorname{sen}(t) \cdot \underbrace{\cancel{\cos\frac{3\pi}{2}}}_{\to 0} + \underbrace{\operatorname{sen}\frac{3\pi}{2}}_{\to -1} \cdot \cos t \right\}$$

$$F(S) = -e^{-\frac{3\pi s}{2}} \mathcal{L}\left\{ \cos(t) \right\}$$

$$F(S) = -e^{-\frac{3\pi s}{2}} \left(\frac{s}{s^2+1} \right)$$

b) $f(t) = \begin{cases} \operatorname{sen} t & 0 \leq t < 3\pi/2 \\ 0 & 3\pi/2 \leq t \end{cases}$

$$f(t) = \operatorname{sen}(t) \left(1 - U_{3\pi/2}\right)$$

$$f(t) = \operatorname{sen} t - U_{\frac{3\pi}{2}} \operatorname{sen}(t)$$

$$\mathcal{L}\{f(t)\} = F(s)$$

$$F(S) = \frac{1}{s^2+1} - e^{-\frac{3\pi s}{2}} \mathcal{L}\left\{ \operatorname{sen}\left(t + 3\pi/2\right) \right\}$$

$$F(S) = \frac{1}{s^2+1} + e^{-\frac{3\pi s}{2}} \left(\frac{s}{s^2+9} \right)$$

(3) $e^{at} f(t) = F(t)\Big|_{S=S-a}$

$a = Ki$

$\mathcal{L}\{t e^{Kti}\}$ Puede usarse para deducir

$\mathcal{L}\{t\cos(Kt)\} = \dfrac{S^2-K^2}{(S^2+K^2)^2}$

$\mathcal{L}\{t\,\text{sen}(Kt)\} = \dfrac{2SK}{(S^2+K^2)^2}$

$e^{Kti} = \cos(Kt) + i\,\text{sen}(Kt)$

$\mathcal{L}\{t\cos(Kt) + it\,\text{sen}(Kt)\}$

$\dfrac{1}{S^2}\Big|_{S=S-Ki}$

$\dfrac{1}{(S-Ki)^2} \cdot \dfrac{(S+Ki)^2}{(S+Ki)^2}$

$\dfrac{(S+Ki)^2}{[S^2-(Ki)^2]^2}$ $\dfrac{S^2+2SKi+(Ki)^2}{(S^2+K^2)^2}$

$$\frac{s^2-K^2}{(s^2+K^2)^2} + i\,\frac{2Ks}{(s^2+K^2)^2}$$

$$\frac{s^2-K^2}{(s^2+K^2)^2} \rightarrow t\cos(Kt) \qquad \frac{2Ks}{(s^2+K^2)^2} \rightarrow t\,\mathrm{sen}(Kt)$$

(4) Encuentre $\mathcal{L}^{-1}\{F(s)\}$

(a) $F(s) = \frac{2s+1}{s^2-2s+2}$

$$F(s) = \frac{2s+1}{(s^2-2s+1)+1} = \frac{2s+1}{(s-1)^2+1} = \frac{(s-1)+s+2}{(s-1)^2+1}$$

$$= \frac{(s-1)}{(s-1)^2+1} + \frac{(s-1)}{(s-1)^2+1} + \frac{3}{(s-1)^2+1}$$

$$= e^t\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} + e^t\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} + e^t\mathcal{L}^{-1}\left\{\frac{3}{s^2+1}\right\}$$

$$= e^t\cos(t) + e^t\cos(t) + 3e^t\,\mathrm{sen}(t)$$

$$f(t) = e^t\left(2\cos(t) + 3\,\mathrm{sen}(t)\right)$$

(b) $F(s) = \frac{1-2s}{s^2+4s+5}$

$$\frac{1-2s}{(s+2)^2+1} = \frac{1-2(s+2-2)}{(s+2)^2+1} = \frac{1-2(s+2)+4}{(s+2)^2+1}$$

$$\frac{5-2(s+2)}{(s+2)^2+1}$$

$$\frac{5}{(s+2)^2+1} - 2\,\frac{(s+2)}{(s+2)^2+1}$$

$$5e^{-2t}\,\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} - 2e^{-2t}\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\}$$

$$5e^{-2t}\,\text{sen}(t) - 2e^{-2t}\cos(t)$$

ⓒ $F(s) = \dfrac{8s^2-4s+12}{s(s^2+4)}$

$$\frac{8s^2-4s+12}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+D}{(s^2+4)}$$

$$8s^2-4s+12 = A(s^2+4) + s(Bs+D)$$

$s=0$

$12 = A4$

$\boxed{A=3}$

$s=2i$

$-32-8i+12 = 2i(2Bi+D)$

$-8i-20 = -4B+2iD$

$-8 = 2D$ $\quad\boxed{D=-4}$

$-20 = -4B$

$\boxed{B=5}$

$$\mathcal{L}^{-1}\left\{\frac{3}{s} + \frac{5s}{s^2+4} - \frac{4}{s^2+4}\right\}$$

$$3 + 5\cos(2t) - 2\,\text{sen}(2t)$$

d) $F(s) = \dfrac{(s+1)^2}{(s+2)^4} = \dfrac{(s+1)^2}{(s+2)^4} = \dfrac{A}{(s+2)} + \dfrac{B}{(s+2)^2} + \dfrac{C}{(s+2)^3} + \dfrac{D}{(s+2)^4}$

$$(s+1)^2 = A(s+2)^3 + B(s+2)^2 + C(s+2) + D$$

$s = -2$

$\boxed{1 = D}$

Derivamos

$$2(s+1) = 3A(s+2)^2 + 2B(s+2) + C$$

$s = -2$

$\boxed{-2 = C}$

Derivamos

$2(1) = 6A(s+2) + 2B$ $\qquad 0 = 6A$

$2 = 6A(s+2) = 2B$ $\qquad \boxed{A = 0}$

$s = -2$

$2 = 2B$

$\boxed{B = 1}$

e) $F(s) = \dfrac{2s-4}{(s^2+s)(s^2+1)}$

$$\frac{2s-4}{s(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{Cs+D}{(s^2+1)}$$

$$2s-4 = A(s+1)(s^2+1) + s(s^2+1)B + s(s+1)(Cs+D)$$

$s=-1$

$-2-4 = -2B$

$-6 = -2B$

$B=3$

$s=0$

$\boxed{-4=A}$

$s=i$

$2i-4 = 0+0+(-1+i)(Ci+D)$

$\cancel{2i}-\cancel{4} = -Ci - D - C + Di$

$2 = -C + D$

$-4 = -D - C$

$-2 = -2C$

$\boxed{C=1}$

$2 = -1 + D$

$\boxed{D=3}$

$$\mathcal{L}^{-1}\left\{\frac{-4}{s} + \frac{3}{s+1} + \frac{s}{s^2+1} + \frac{3}{s^2+1}\right\}$$

$y(t) = -4 + 3e^{-t} + \cos(t) + 3\operatorname{sen}(t)$

(F) $F(s) = \frac{-2}{4s+5}$

$$\mathcal{L}^{-1}\left\{\frac{-2}{4(s+5/4)}\right\}$$

$$-\frac{1}{2} \cdot \frac{1}{s+5/4}$$

$$\frac{-e^{-5t/4}}{2}$$

(G) $F(s) = \frac{2s-5}{3s^2+12}$

$$\mathcal{L}^{-1}\left\{\frac{2s-5}{3(s^2+4)}\right\}$$

$$\frac{1}{3}\mathcal{L}^{-1}\left\{\frac{2s}{s^2+4} - \frac{5}{s^2+4}\right\}$$

$$\frac{1}{3}\left(2\cos(2t) - \frac{5}{2}\operatorname{sen}(2t)\right)$$

(H) $F(s) = \frac{(s-2)e^{-s}}{s^2-4s+3}$ $\quad \frac{(s-2)e^{-s}}{s^2-4s+4-1}$

$$\mathcal{L}^{-1}\left\{\frac{(s-2)e^{-s}}{(s-2)^2-1}\right\}$$

$$e^{2t}\, u_1(t)\, \cosh(t)$$

$$\boxed{e^{2t-1}\, u_1(t)\, \cosh(t-1)}$$

~~I $F(s) = 2(s-1)\, e^{-2s}$ / $(s^2+s$~~

I $F(s) = \dfrac{2\,(s-1)\,e^{-2s}}{s^2-2s+2}$

$$\frac{2\,(s-1)\,e^{-2s}}{s^2-2s+1+1} \quad \mathcal{L}^{-1}\left\{\frac{2\,(s-1)\,e^{-2s}}{(s-1)^2+1}\right\}$$

$$2\,U_2(t)\,e^{t} \quad \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\}$$

$$2\,U_2(t)\,e^{t}\,\cos(t)$$

$$2\,U_2(t)\,e^{(t-2)}\,\cos(t-2)$$

J $\dfrac{e^{-s}+e^{-2s}-e^{-3s}-e^{-4s}}{s}$

$$\frac{e^{-s}}{s}+\frac{e^{-2s}}{s}-\frac{e^{-3s}}{s}-\frac{e^{-4s}}{s}$$

$$U_1(t)+U_2(t)-U_3(t)-U_4(t)$$

(K) $F(S) = \dfrac{e^{-\pi s}}{s^3}$

$U_\pi(t) \cdot \dfrac{1}{2} t^2$

$\dfrac{U_\pi(t)}{2} (t-\pi)^2$

(L) $F(S) = \dfrac{e^{-\pi s/2}}{s^2+4}$

$U_{\pi/2} \quad \dfrac{1}{2} \quad \mathcal{L}^{-1} \left\{ \dfrac{2}{s^2+4} \right\}$

$\dfrac{U_{\pi/2}}{2} \quad \text{sen}(2t)$

$U_{\pi/2}(t) \quad \dfrac{\text{sen}(2(t-\pi/2))}{2}$

$U_{\pi/2}(t) \quad \text{sen}(2t-\pi)$

5

(a) $y'' - 2y' + 2y = \cos(t)$

, $y(0) = 1$
$y'(0) = 0$

$y'' - 2y' + 2y = \cos(t)$

$\mathcal{L}\{y(t)\} = Y(s)$

$\mathcal{L}\{y''\} - 2\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = \mathcal{L}\{\cos(t)\}$

$s^2 Y(s) - s\,\underbrace{y(0)}_{\to 1} - \underbrace{y'(0)}_{\to 0} - 2s\,Y(s) + 2\,\underbrace{y(0)}_{\to 1} + 2\,Y(s) = \dfrac{s}{s^2+1}$

$s^2 (Y(s)) - s - 2s\,Y(s) + 2 + 2\,Y(s) = \dfrac{s}{s^2+1}$

$Y(s)\left[s^2 - 2s + 2\right] = \dfrac{s}{s^2+1} + s - 2$

$(s^2-2s+2)\,Y(s) = \dfrac{s^3 + 2s - 2s^2 - 2}{s^2+1}$

$Y(s) = \dfrac{s^3 + 2s - s^2 - 2}{(s^2-2s+2)} \cdot \dfrac{1}{s^2+1}$

$$\begin{array}{l|l} s^3 - 2s^2 + 2s - 2 & s^2-2s+2 \\ \cline{2-2} -s^3 + 2s^2 - 2s & s \\ \hline \qquad\qquad -2 & \end{array}$$

$\left[s - \dfrac{2}{s^2-2s+2}\right] \dfrac{1}{s^2+1}$

$\dfrac{s}{s^2+1} - \dfrac{2}{(s^2+1)(s^2-2s+2)}$

$\to$ FP

$\mathcal{L}^{-1}\{Y(s)\} = y(t)$

$$y(t) = \cos(t) - \mathcal{L}^{-1}$$

ⓑ $y'' + 4y = U_\pi - U_{2\pi}$

$y(0) = 1$ $\qquad \mathcal{L}\{y(t)\} = Y(s)$

$y'(0) = 0$

$$\mathcal{L}\{y''\} + \mathcal{L}\{4y\} = \mathcal{L}\{U_\pi - U_{2\pi}\}$$

$$s^2 Y(s) - s\,y(0)^{\to 1} - y'(0)^{\to 0} + 4\,Y(s) = \frac{e^{-\pi s}}{s} - \frac{e^{-2\pi s}}{s}$$

$$Y(s)\left[s^2 + 4\right] = \frac{e^{-\pi s}}{s} - \frac{e^{-2\pi s}}{s} + s$$

$$Y(s) = \frac{e^{-\pi s}}{s(s^2+4)} - \frac{e^{-2\pi s}}{s(s^2+4)} + \frac{s}{s^2+4}$$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t) \qquad \frac{\frac{1}{4}}{s} - \frac{s/4}{s^2+4}$$

$$y(t) = U_\pi(t)\left(\frac{1}{4} - \frac{1}{4}\cos(2t+2\pi)\right)$$

$$- U_{2\pi}\left(\frac{1}{4} - \frac{1}{4}\cos(2t+4\pi)\right) + \cos(2t$$

$$y(t) = \frac{1}{4}\left[U_\pi (1 - \cos(2t)) - U_{2\pi}(1 - \cos(2t)) \right] + \cos(2t)$$

ⓒ $\begin{cases} y'' + y' + \frac{5}{4} y = \begin{cases} \text{sen } t & 0 \le t < \pi \\ 0 & \pi \le t \end{cases} \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$

$\hookrightarrow \text{sen}(t)(1 - U_\pi)$

$\text{sen}(t) - U_\pi \text{ sen}(t)$

$\mathcal{L}\{y(t)\} = Y(s)$

$\mathcal{L}\{y''\} + \mathcal{L}\{y'\} + \frac{5}{4}\mathcal{L}\{y\} = \mathcal{L}\{\text{sen}(t)\} - \mathcal{L}\{U_\pi \text{ sen}(t)\}$

$s^2 Y(s) - \cancel{s\, y(0)}^{\to 0} - \cancel{y'(0)}^{\to 0} + s Y(s) - \cancel{y(0)}^{\to 0} + \frac{5}{4} Y(s) = \frac{1}{s^2+1} - e^{-\pi s} \mathcal{L}\{\text{sen}(t+\pi)\}$

$Y(s)\left[s^2 + s + 5/4\right] = \frac{1}{s^2+1} - e^{-\pi s} \mathcal{L}\{ \text{sen}(t) \cdot \cos\pi + \cancel{\text{sen}(\pi) \cos(t)}^{\to 0}$

$Y(s)\left[s^2 + s + 5/4\right] = \frac{1}{s^2+1} + e^{-\pi s} \mathcal{L}\{\text{sen}(t)\}$

$Y(s)\left[s^2 + s + 5/4\right] = \frac{1}{s^2+1} + \frac{e^{-\pi s}}{s^2+1}$

$$\frac{As+B}{s^2+1} + \frac{\quad}{(s+\frac{1}{2})^2 + 1}$$

(d) $\begin{cases} y'' + 4y = \begin{cases} t & 0 \le t < \pi \\ 1 & \pi \le t \end{cases} \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$

$t(1 - U_{\pi}) + U_{\pi}$

$t - tU_{\pi} + U_{\pi}$

$t + U_{\pi}(-t + 1)$

$\mathcal{L}\{y(t)\} = Y(s)$

$\mathcal{L}\{y''\} + 4\mathcal{L}\{y\} = \mathcal{L}\{t\} + e^{-\pi s}\mathcal{L}\{1 - (t + \pi)\}$

$s^2 Y(s) - \cancel{sy(0)}^{\to 0} - \cancel{y'(0)}^{\to 0} + 4Y(s) = \frac{1}{s^2} + e^{-\pi s}\mathcal{L}\{1 - t - \pi\}$

$Y(s)\,[s^2 + 4] = \frac{1}{s^2} + e^{-\pi s}\left[\cancel{\frac{1}{s}} - \frac{1}{s^2} - \cancel{\frac{1}{s}}\right]$

$Y(s) = \frac{1}{s^2(s^2+4)} - \frac{e^{-\pi s}}{s^2(s^2+4)}$

$\frac{\frac{1}{4}}{s^2} - \frac{\frac{1}{4}}{s^2+4}$

$Y(s) = \frac{1}{4}\left[\frac{1}{s^2} - \frac{1}{s^2+4}\right] - \frac{e^{-\pi s}}{4}\left[\frac{1}{s^2} - \frac{1}{s^2+4}\right]$

$$Y(t) = \mathcal{L}^{-1}\{Y(s)\}$$

$$Y(t) = \frac{1}{4}\left[t - \frac{1}{2}\operatorname{Sen}(2t)\right] - \frac{U_{\pi}}{4}\left[t - \pi - \frac{1}{2}\operatorname{Sen}(2t - 2\pi)\right]$$

(6) Suponga que $F(s) = \mathcal{L}\{f(t)\}$ existe para $s > a \geq 0$

(a) Demuestre que si se es una constante positiva entonces

$$\mathcal{L}\{f(ct)\} = \frac{1}{c} F\left(\frac{s}{c}\right) \qquad s > ca$$

$$\int_0^{\infty} e^{-st} f(ct)\, dt \qquad \tau = ct$$

$$c\,dt = d\tau$$

$$\int_0^{\infty} e^{-s\tau/c} f(\tau) \frac{d\tau}{c} \qquad \frac{d\tau}{c} = dt$$

$$\frac{1}{c}\int_0^{\infty} e^{-s\tau/c} f(\tau) = \frac{1}{c} F(s/c)$$

(b) Demuestre que si K es una constante positiva entonces

$$\mathcal{L}^{-1}\{F(ks)\} = \frac{1}{k} f\left(\frac{t}{k}\right)$$

$$F(ks) = \frac{1}{K}\mathcal{L}\left\{f\left(\frac{t}{k}\right)\right\}$$

$$c = \frac{1}{k}$$

$F(\cancel{k}s) = c \; \mathcal{L}\{F(ct)\}$

$\frac{1}{c} F\left(\frac{s}{c}\right) = \mathcal{L}\{F(tc)\}$

(C) ~~[illegible]~~

Demuestre que si α y β son constantes positivas entonces

$$\mathcal{L}^{-1}\{F(\alpha s+\beta)\} = \frac{1}{\alpha} e^{-\frac{\beta t}{\alpha}} f\left(\frac{t}{a}\right)$$

$$\mathcal{L}^{-1}\{F(\alpha(s+\beta/\alpha))\}$$

(7)

a) $F(s) = \frac{2^{n+1} n!}{s^{n+1}} \quad t^n$

$$\mathcal{L}^{-1}\{F(s)\} = \frac{1}{2}^{n+1}\left\{\frac{n!}{s^{n+1}}\right\}$$

$$\boxed{Y(t) = 2^{n+1} \; t^n}$$

b) $F(s) = \frac{2s+1}{4s^2+4s+5}$

$F(s) = \frac{1}{2} \frac{(s+1/2)}{s^2+s+5/4}$

$F(s) = \frac{1}{2} \frac{(s+1/2)}{(s+1/2)^2+1}$

$\mathcal{L}\{F(s)\} = Y(t)$

$Y(t) = \mathcal{L}^{-1}\left\{\frac{s+1/2}{(s+1/2)^2+1}\right\}$

$Y(t) = \frac{1}{2} e^{-t/2} \cos(t/2)$

c) $F(S) = \dfrac{e^2 e^{-4S}}{2S-1}$

$\mathcal{L}^{-1}\left\{\dfrac{e^{-4S+2}}{2S-1}\right\} = \dfrac{\mathcal{L}^{-1}}{2}\left\{\dfrac{e^{-4(S-1/2)}}{(S-1/2)}\right\}$

$\dfrac{e^{t/2}}{2} \mathcal{L}^{-1}\left\{\dfrac{e^{-4S}}{S}\right\}$

$\dfrac{e^{t/2}}{2} U_4(t/2)$

8) Use $\int_0^t F(\tau)\,d\tau = \mathcal{L}^{-1}\left\{\dfrac{F(S)}{S}\right\}$

a) $\mathcal{L}^{-1}\left\{\dfrac{1}{S(S-1)}\right\}$

$\mathcal{L}^{-1}\left\{\dfrac{\frac{1}{(S-1)}}{S}\right\}$ e^t

$\int_0^t e^{\tau}\,d\tau \ast 1$

$e^{\tau}\Big|_0^t$ $(e^t - e^0)$ $\boxed{(e^t - 1)}$

(b) $\mathcal{L}^{-1}\left\{\frac{1}{s^2(s-1)}\right\}$

$$\frac{\frac{1}{s(s-1)}}{s} \qquad \frac{\frac{\frac{1}{(s-1)}}{s}}{s}$$

e^t

$e^t - 1$

$$\int_0^t e^\tau - 1 \, d\tau$$

$$\int_0^t e^\tau - \int_0^t 1 \, d\tau$$

$$(e^t - 1) - (t)$$

$$te^t - t$$

$$\mathcal{L}^{-1} = \left\{\frac{1}{s^2(s-1)}\right\} = \boxed{te^t - t}$$

(c) $\mathcal{L}^{-1}\left\{\frac{1}{s^3(s-1)}\right\}$

$$\frac{\frac{1}{s^2(s-1)}}{s}$$

$te^t - t$

$$\int_0^t \tau e^\tau \, d\tau - \int_0^t \tau \, d\tau$$

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$u = \tau \qquad dv = e^{\tau}\, d\tau$

$du = d\tau \qquad v = e^{\tau}$

$$\tau e^{\tau}\Big|_0^t - \int_0^t e^{\tau}\, d\tau$$

$$\tau e^{t}\Big|_0^t - e^{\tau}\Big|_0^t$$

$$t e^{t} - 0(e^{0}) - (e^{t} - 1)$$

$$t e^{t} - e^{t} + 1$$

$$\mathcal{L}^{-1}\left\{ \frac{1}{s^2(s-1)} \right\} = \boxed{e^{t}(t-1) + 1}$$

9

$P = m \cdot g$

$\frac{\cancel{4}}{\cancel{32}} = m$ (2, 1 / 16)

$\boxed{m = 1/8}$

$\frac{\cancel{9}}{\cancel{12}}$ ... $\frac{3}{2} =$

$4 = k \cdot 2$

$\boxed{k = 2}$

$\beta = \frac{7}{8}$

$\beta = \frac{7}{8}$

$x(0) = -3/2$

$x'(0) = 0$

$$\frac{x''}{8} + x'\frac{7}{8} + 2x = 0$$

$$x'' + 7x' + 16x = 0$$

$$\mathcal{L}\{y(t)\} = Y(s)$$

$$\mathcal{L}\{x''\} + 7\,\mathcal{L}\{x'\} + 16\,\mathcal{L}\{x\} = 0$$

$$s^2 X(s) - s\,x(0) - \cancel{x'(0)} \xrightarrow{} 0 + 7s X(s) - x(0) + 16\,X(s) = 0$$

$$X(s)\left(s^2 + 7s + 16\right) = -\frac{3}{2}s - \frac{3}{2}$$

$$X(s) = -\frac{3}{2}\left(\frac{s}{s^2+7s+16} + \frac{1}{s^2+7s+16}\right)$$

$$\left(\frac{7}{2}\right)^2 \qquad \frac{49}{4}$$

$$X(s) = -\frac{3}{2}\left(\frac{s}{(s+7/2)^2+15/4} + \frac{1}{(s+7/2)^2+15/4}\right)$$

$$\mathcal{L}^{-1}\{X(s)\} = x(t)$$

$$-\frac{3}{2}e^{-\frac{7t}{2}}\mathcal{L}\left\{\frac{s-7/2}{s^2+15/4} + \frac{1}{s^2+15/4}\right.$$

$$-\frac{3}{2}e^{-\frac{7t}{2}}\left[\mathcal{L}\left\{\frac{s}{s^2+15/4}\right\}-\frac{7}{4}\mathcal{L}\left\{\frac{1}{s^2+15/4}\right\}\right.$$

$$\left.+\mathcal{L}\left\{\frac{1}{s^2+15/4}\right\}\right]$$

$$-\frac{3}{2}e^{-\frac{7t}{2}}\left[\cos\left(\frac{\sqrt{15}}{2}t\right)-\frac{7}{\cancel{4}}\frac{\cancel{4}}{\sqrt{15}}\,\text{sen}\left(\frac{\sqrt{15}}{2}t\right)\right.$$

$$\left.+\frac{4}{\sqrt{15}}\,\text{sen}\left(\frac{\sqrt{15}}{2}t\right)\right]$$

10) 32 lb

$\boxed{m = 1 \text{ slug}}$

$\delta = 2$

$32 = K\delta$

$\boxed{K = 16}$

$$f_e(t)=\begin{cases}20t & 0\le t\le 5\\ 0 & 5\le t\end{cases}$$

$\beta = 0$

$x(0) = 0$

$x'(0) = 0$

$$x'' + 16x = 20t\,(1 - U_5(t))$$

$$x'' + 16x = 20t - 20t\, U_5(t)$$

$$\mathcal{L}\{x(t)\} = \bar{x}(s)$$

$$\mathcal{L}\{x''\} + 16\,\mathcal{L}\{x\} = 20\,\mathcal{L}\{t\} - \mathcal{L}\{U_5(t)\,(20t)\}$$

$$s^2 x(s) - s\,x(0)^{\to 0} - x'(0)^{\to 0} + 16\,x(s) = \frac{1}{s^2} - e^{-5s}\,\mathcal{L}\{20(t+5)\}$$

$$x(s)\left[s^2 + 16\right] = \frac{1}{s^2} - e^{-5s}\left[\mathcal{L}\{20t + 100\}\right]$$

$$x(s)\left[s^2+16\right] = \frac{1}{s^2} - e^{-5s}\left[\frac{20}{s^2} + \frac{100}{s}\right]$$

$$x(s) = \frac{1}{s^2(s^2+16)} - \frac{20e^{-5s}}{s^2} - \frac{e^{-5s}\,100}{s}$$

$$\mathcal{L}^{-1}\{x(s)\} = x(t)$$

$$x(t) \overset{\mathcal{L}^{-1}}{=} \Big\{ \frac{1}{16}\,\frac{1}{s^2} - \frac{1}{16}\,\frac{1}{s^2+16} - \frac{20e^{-5s}}{s^2} - \frac{e^{-5s}\,100}{s}$$

$$x(t) = \frac{t}{16} - \frac{1}{16 \cdot 4} \operatorname{Sen}(4t) - 20\, U_5(t)\,(t-5) - 100\, U_5(t)$$

$$x(t) = \frac{1}{16}\left[t - \frac{\operatorname{Sen}(4t)}{4}\right] + U_5(t)\left[-20(t-5) - 100\right]$$